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THE POINCARÉ METHOD IN TEACHING THE THEORY OF NONLINEAR VIBRATIONS

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Annotations. In the realm of continuum mechanics, Poincaré's theory provides a framework for understanding the nonlinear propagation of waves in diverse media, such as nonlinear acoustic waves, solitary waves (solitons), and shock waves. By characterizing the stability, bifurcations, and interactions of nonlinear waves, researchers can elucidate the underlying mechanisms governing wave phenomena

and predict complex wave behavior in heterogeneous materials. Poincaré's theory finds application in the analysis of fluid-structure interaction (FSI) problems, where the dynamic coupling between fluid flow and structural deformation leads to nonlinear oscillatory behavior. By considering the mutual influence of fluid forces and structural response, Poincaré's methods enable the prediction of resonant phenomena, vortex-induced vibrations, and instability mechanisms in FSI systems, with implications for engineering design and optimization.

Keywords: Poincaré method, nonlinear oscillations, dynamical systems, Chaos theory, nonlinear dynamics, mathematical physics, differential equations, educational technology

Introduction: We proved that the Poincaré's nonlinear oscillation theory can be extended to the continuum mechanics and suggested a method of direct perturbation of partial differential equation. and weighted integration to calculate the resonant and nonresonant periodic solutions of the continuum system. In this paper by using the above method we calculate some examples. These examples show that our method is effective in application.

In recent years many authors used the perturbation method of partial differential equation to solve the vibration problem of continuum mechanics. In [2] Keller and Ting discussed the wave and vibration problem for the nonlinear medium with infinitive large dimension, and they called their method improved perturbation one. As indicated in [3]. "The main thought of Keller and Tings method is as follows: Perturb the parameter, multiply the perturbed nonhomogenous equation by weight function and integrate it (usually choose the solution of corresponding homogeneous equation as weight function), then get the solvable conditions by which we can define the deformed parameter."

But the method in our paper is quite different from that given by Keller and Ting. Our method is based upon the Poincaré's theory and is mainly used to calculate the resonant and nonresonant periodic solution of continuum medium. The principal thought of our method is: In resonant case in order to avoid the small denominator

term to appear in the periodic solution of linear derivative equation, we introduce a parameter resolution method, by which we can transform the partial differential equation for forced vibration into such a form that the terms standing at the right side can be reduced to small quantity and then be merged into the equation of higher order. After that by using the condition of periodicity and weighted integration, we can determine the constants within the derivative periodic solution.

This method has some characteristics.

(1) The result obtained is complete. Formerly the periodic solution of nonlinear vibration for beams, thin plates and shells are calculated as follows: Substitute the space function satisfying

boundary conditions into the partial differential equation of continuum system, use the approximate method (such as Galerkin method) to reduce the partial differential equation to nonlinear ordinary differential equation, then find out the solution by means of small parameter method, average method or multiple scale method. Therefore the results obtained depend upon the choice of space function and thus the understanding of nonlinear oscillation of continuum system is restricted. But our method does not have the above shortage. Our method is based upon the Poincaré's theory. After the partial differential equation being directly perturbed, the solution of all the perturbed equations can be expanded into generalized Fourier series in eigenfunctions of derivative system. Under certain conditions we can determine the proportion of each eigenfunction and decide which should be retained and which should be given up in the solution, therefore the result obtained by our method is complete. In order to differentiate our method from the others we call it direct perturbation method of partial differential equation.

(2) Because the solution of all the perturbed equations can be expanded into generalized Fourier series in eigenfunctions of derivative system, we can give a unified formula for calculating the periodic solution of such problems which have same geometric shape and constitutional property but different boundary conditions and load distribution. Hence it is convenient in application.

(3) The operations included in calculating the resonant and nonresonant periodic solution are only the integration of time function and avoid solving nonlinear differential equation. Therefore in solving process we only need to concentrate our attention on discussion of boundary value problem. Combining our method with proper approximate method of boundary value problem we can effectively solve problems which have very complex boundary conditions.

II. The Forced Vibration of Elastic Beam with Fixed Span

The differential equations of elastic beam with fixed span and under the action of transverse load are

$$EI \frac{\partial^4 \omega^*}{\partial x^{*4}} + \mu \frac{\partial^2 \omega^*}{\partial t^{*2}} = EF \frac{\partial}{\partial x^*} \left[\frac{\partial \omega^*}{\partial x^*} \frac{\partial v^*}{\partial x^*} + \frac{1}{2} \left(\frac{\partial \omega^*}{\partial x^*} \right)^2 \right] + 2\lambda^* \frac{\partial \omega^*}{\partial t^*} + q^*(x^*, t^*) \quad (1)$$

$$\mu \frac{\partial^2 \omega^*}{\partial t^{*1}} = EF \frac{\partial}{\partial x^*} \left[\frac{\partial v^*}{\partial x^*} + \frac{1}{2} \left(\frac{\partial \omega^*}{\partial x^*} \right)^1 \right] \quad (2)$$

$$S^* = EF \left[\frac{\partial v^*}{\partial x^*} + \frac{1}{2} \left(\frac{\partial \omega^*}{\partial x^*} \right)^1 \right] \quad (3)$$

where ω^* is deflection of beam, v^* is transverse displacement, S^* is inner axial force, μ is mass per longitudinal length, F is area of transverse section, E is elastic modulus, λ^* is damping coefficient.

According to Kirchhoff's assumption, the longitudinal inertia force in (2) can be neglected. Introduce the following dimensionless quantities:

$$\begin{aligned} x &= \frac{x^*}{l}, & t &= \sqrt{\frac{EL}{\mu l^4}} t^*, & \omega &= \frac{l}{h^2} \omega^* \\ v &= \frac{l^3}{h^4} v^*, & S &= \frac{l^4}{EFh^4} S^*, & q &= \frac{l^6}{EIh^2} q^* \end{aligned}$$

$$v = \frac{Fh^2}{12I}, \quad \lambda = \frac{l^4 \lambda^*}{12h^2 \sqrt{\mu EI}}, \quad \varepsilon = \frac{12h^2}{l^2}$$

in which l is span of beam, h is height of beam, then (1), (2) and (3) can be reduced to

$$\frac{\partial^4 \omega}{\partial x^4} + \frac{\partial^2 \omega}{\partial t^2} = q(x, t) - 2e\lambda \frac{\partial \omega}{\partial t} + \varepsilon v S \frac{\partial^2 \omega}{\partial x^2} \quad (4)$$

$$S = \frac{\partial v}{\partial x} + \frac{1}{2} \left(\frac{\partial \omega}{\partial x} \right)^2 \quad (5)$$

$$\frac{\partial S}{\partial x} = 0 \quad (6)$$

Suppose $q(x, t)$ is a periodic function in with a period $\frac{2\pi}{\Omega}$ i.e.

$$q(x, t) = q\left(x, t + \frac{2\pi}{\Omega}\right)$$

and the boundary conditions are

$$v(0) = v(1) = 0 \quad (7)$$

$$w(0) = w(1) = 0 \quad (8)$$

$$\left. \frac{\partial^4 \omega}{\partial x^4} \right|_{n=0} = \left. \frac{\partial^f \omega}{\partial x^f} \right|_{n=1} = 0 \quad (i, j = 1, 2) \quad (9)$$

both ends are fixed as $i = j = 1$, simply supported as $i = j = 2$, one fixed and other simply supported as

$$w(x, t) = w_0(x, t) + ew_1(x, t) + e^2 w_2(x, t) + \dots \quad (10)$$

$$v(x, t) = v_0(x, t) + ev_1(x, t) + e^2 v_2(x, t) + \dots \quad (11)$$

$$S(t) = S_0(t) + eS_1(t) + e^2 S_2(t) + \dots \quad (12)$$

Substitute (10), (11), (12) into (4), (5), equate the coefficients of of the same order on both sides, then we get

$$\frac{\partial^4 \omega_0}{\partial x^4} + \frac{\partial^2 \omega_0}{\partial t^2} = q(x, t) \quad (13)$$

$$\frac{\partial^4 \omega_1}{\partial x^4} + \frac{\partial^2 \omega_1}{\partial t^2} = v S_0(t) \frac{\partial^2 \omega_0}{\partial x^2} - 2\lambda \frac{\partial \omega_0}{\partial t} \quad (14)$$

$$S_0(t) = \frac{\partial v_0}{\partial x} + \frac{1}{2} \left(\frac{\partial \omega_0}{\partial x} \right)^2 \quad (15)$$

Therefore for thin plate (note $e = 12(1 - \nu^2)h^2/l^2$, so the smaller h/l , the smaller e) and small damping (smaller A), as is close to one of lower intrinsic frequency of derivative system, there would be $\varepsilon\lambda\Omega \ll 1$. On the other hand, as mentioned above, the nonresonant term in (3) is a small quantity and is independent upon e and A . Thus under the above conditions in calculating resonant periodic solution it is permitted to neglect the nonresonant terms. Conversely, for thick plate, large damping and high frequency resonance ($\Omega \approx \omega_{rr}, r \gg 1$), the resonant term and nonresonant term are of the same order, so in calculation of resonant periodic solution the nonresonant term are not negligible. If we adopt the current approximate method to calculate the above problem, the procedure is

as follows: Substitute $w(x, y, t) = g(t) \sin x \sin y$ into (5), obtain (x, y, t) ; substitute $w(x, y, t)$ and $p(x, y, t)$ into (4); by using Galerkin method reduce (4) into nonlinear ordinary differential equation, from which we get the 1st order approximate solution; finally by solvable condition we obtain two equations for determining M and N , which are the same as (3) and (4). It is clear the above procedure is more complicated than that of ours.

As $N > 1$ (superharmonic resonance), the equations for determining M and N are as follows

$$\begin{cases} (aM^2 - \sigma)M \cos \theta - 2\lambda MN \sin \theta = 0 \\ (aM^2 - \sigma)M \sin \theta + 2\lambda MN \cos \theta = 0 \end{cases}$$

(6) and (7) have only the zero solution $M=0$, obviously it is not in agreement with the fact. This is owing to the fact that we neglect the nonresonant terms in (8) which originally should not. In reality in superharmonic resonance the resonant and nonresonant terms are of the same order, their 2nd derivatives are of the same order too. Therefore when we neglect the nonresonant terms, as a result the M obtained by (6) and (7) would be zero. Thus $M=0$ does not mean no superharmonic resonance to exist, it merely means the amplitude of superharmonic resonance is much smaller and is of the same order as the nonresonant term.

(B) Double mode resonance $q=0$

we still study the principal resonance of lower frequency for thin plate with small damping. Suppose

$$w_0(x, y, t) = M_1 \cos(\Omega t + \theta_1) \sin r\pi x \cdot \sin r\pi y + M_2 \cos(\Omega t + \theta_2) \sin r\pi x \cdot \sin r\pi y$$

in which ser. Substituting (8) into (9) and integrating it, we have

If we exchange M_1 and M_2 , and so θ_1 and θ_2 , and the form of (4) remains unchanged, thus we know $M^1 = M_2 = \tilde{M}$, $\theta_1 = \theta_2 = \tilde{\theta}$ Eliminating $\tilde{\theta}$ we get

$$\sigma = (\beta + \gamma)\tilde{M}^2 \pm \sqrt{\frac{q_1^2}{e^2\tilde{M}^2} - 4\gamma^2\Omega^2}$$

If in expressions for σ and y we put sr , it yields $B + y = 4a$. After that we add the 1st equation of (4) to the 3rd, the 2nd to the 4th, and put $M_1 = M_2 = \tilde{M}$, $\theta_1 = \theta_2 = \tilde{\theta}$ then again we obtain (3) and (4).

Since $\xi_{rr} > 0$ from (5) and (6) it can be seen under simply supported conditions, the amplitude-frequency curve for $q_0 \neq 0$ (i.e. static load $\neq 0$) would lie in the right side of that for $q_0 = 0$ (i.e. static load=0), this means that the static load plays a role to raise the frequency of principal resonance. In addition, from (4) and (7) we know that the difference of σ is $\Delta\sigma = \xi_{rr}q_0^2$. Thus as $e\Delta\sigma = O(w_{rr}^2)$, i.e. as

$$q_0 = O\left(\sqrt{\frac{w_{rr}^2}{e\xi_{rr}}}\right)$$

the influence of load average (static load) upon the frequency of principal resonance should be taken into consideration. If we denote the frequency raised by static load by, then from (2) we get

$$\tilde{\omega}_{rr} = \sqrt{\omega_{rr}^2 + e\xi_{rr}q_0^2}$$

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Annotatsiya: XXI asr odamlari ilodkor inson. Maqolada zamonaviy tadqiqotchilarning turli sohalarda kasbiy faoliyat uchun bo‘lajak mutaxassislarni tayyorlash masalalari bo‘yicha fikr keltirilgan. AKT dan foydalanishdagi dars metodologiyasi klassikadan sezilarli farq qilishi haqida bir necha texnologiyalar batafsil bayon qilingan.

Kalit so‘zlar: AKT, raqamli ta'lim manbai, interfaol, seminar, innovatsiya