



## THE PROFILE OF MAGNETIC FLUX DIFFUSION IN SUPERCONDUCTORS

Taylanov Nizom Abdurazzakovich, Eshtuktarova Orzigul Shonazarovna,  
Bebitboyeva Munisa

Jizzax State Pedagogical Institute of Uzbekistan

**Abstract:** The problem of the penetration of a magnetic field into a high-temperature superconductor, which is in the regime of flux creep in an external magnetic field, is considered. Analytical formulas are obtained for the depth and rate of penetration of a magnetic field into a superconductor depending on the values of the problem parameter, namely, on the exponent  $n$  characterizing the rate of penetration of vortices into the superconducting half-space.

**Key words:** flow dynamics, magnetic induction, self-similarity.

Theoretical studies of the patterns of magnetic flux penetration in a various regimes of superconductors were carried out in classical works [1-3]. The dynamics of magnetic flux penetration under the assumption that the differential resistance does not depend on the magnetic field was studied in [2]. In this paper, we consider the nonlinear diffusion problem of the penetration of a magnetic flux into a superconductor taking into account the nonlinear current-voltage characteristic of superconductors, which is valid in the region of low electric fields and in the regime of flux creep. An exact numerical solution is obtained, describing the spatial and temporal evolution of the penetration of the current density, the magnetic and electric fields in the sample. To simulate the process of evolution of small perturbations of the electromagnetic field in space and time, we use the system of equations of macroscopic electrodynamics [3, 4].

The distribution of magnetic induction  $\vec{B}$ , electric field  $\vec{E}$ , and transport current in the superconductor are determined by the following equation

$$\operatorname{rot} \vec{B} = \mu_0 \vec{j}, \quad \operatorname{rot} \vec{E} = \frac{d\vec{B}}{dt}. \quad (1)$$

Using the mathematical formalism developed in [2], we study the influence of differential resistance  $\rho_f(B)$ , on the process of penetration of the magnetic flux the viscous flow regime. The current-voltage characteristic in the regime of viscous flow of vortices can be written in the form

$$\vec{E} = \rho(B) \vec{j}. \quad (2)$$

Here  $\vec{j} = j_c(\vec{B}, T)$ . Combining relation (1) with equation (2), we obtain a nonlinear diffusion equation for the magnetic flux induction  $\vec{B}(\vec{r}, t)$  in the following form

$$\frac{d\vec{B}}{dt} = \frac{1}{\mu_0} \nabla [\rho(B) \nabla \vec{B}]. \quad (3)$$

Obviously, the space-time structure of the solution of the diffusion equation (3) is determined by the nature of the dependence of the differential resistivity on the magnetic field induction  $B$ . Usually in a real experimental situation differential resistance  $\rho(B)$  increases with increasing magnetic field induction

$$\rho(B) = \frac{\Phi_0}{\eta c^2} \vec{B} = \rho_n \frac{\vec{B}}{B_{c2}}, \quad (4)$$



where  $\rho_n$  is the differential resistance in the normal state;  $n$  is the viscosity coefficient,  $B_{c2}$  is the upper critical field of the superconductor. In the case when the differential resistivity  $\rho(B)$  is a linear function of the magnetic field induction  $B$ , the exact solution of the diffusion equation (3) can be easily obtained using known scaling methods [2]. For the complex dependence  $\rho(B)$ , one can use the empirical exponential dependence  $\rho(B) \approx B^n$ , where  $n$  is a positive constant parameter.

Let's consider the evolution of the magnetic flux injected in the infinite thin film (the  $xy$  plane) of a type-II superconductor (the flux lines are perpendicular to the surface). We assume the problem to be homogeneous along  $y$ , so the local magnetic induction  $B$  depends only on the coordinate  $x$  and on time. The current flows along  $y$ . An applied magnetic field is absent. For this dimensional geometry [5], the spatial and temporal evolution of the magnetic field induction is  $B(x, t)$  described by the following nonlinear diffusion equation in a generalized dimensionless form

$$\frac{db}{d\tau} = \frac{d}{d\xi} \left( b^n \left[ \frac{db}{d\xi} \right]^q \right), \quad (5)$$

where we have introduced dimensionless parameters  $b = \frac{B}{B_e}$ ,  $\xi = \frac{\mu_0 j_0}{B_e} x$ ,  $\tau = \frac{t}{t_0}$ ,  $j = \frac{j}{j_0}$ ,  $B_e = \mu_0 j_0 v_0 t_0$

and variables;  $x_p = \frac{B_e}{\rho_0 j_c}$  - is the depth of penetration of the magnetic field in the Bean model;

$t_0 = \rho_n \frac{j_c^2 \mu_0}{B_e^2}$  is the diffusion time;  $q$  is a positive constant parameter. The diffusion equation (5) can

be integrated analytically, taking into account the appropriate initial and boundary conditions at the center of the sample and at its edges. Let us consider the case when the magnetic field applied to the sample increases with time according to a power law with exponent  $\alpha > 0$

$$b(0, t) = b_0 (1+t)^\alpha. \quad (6)$$

$$b(x_p, t) = 0, \quad (7)$$

The boundary condition (5) is equivalent to a linear increase in the magnetic field with time, which corresponds to the real experimental situation. It is easy to see that the case  $\alpha=0$  describes a constant applied magnetic field on the surface of the sample, while the case  $\alpha=1$  corresponds to a linearly increasing applied field, respectively. Here we consider the different cases, namely  $n = 0, 1, 2$  and  $q = 0, 1$ . All examples are computed with  $N = 100$  polynomials for the  $x$  and  $y$ -dependences, and  $Nt = 1000$  time steps. Note that larger values of these parameters have only an effect on the solution below plotting accuracy, i.e., the resulting figures would be indistinguishable from the ones shown. The spatial and temporal profiles of the magnetic flux are shown in Figures 1-3. We first consider the case  $n=0$  and  $q=1$  in Fig. 1, on the left the initial condition, on the right the solution for  $t = 0.5$ . The solution is clearly unstable in the sense that the initial perturbations grow. In addition the simulations were performed for the coefficient  $\alpha=1$ , final time  $t=10$ , time discretization  $M = 91$ , and space discretization  $N = 82$ .

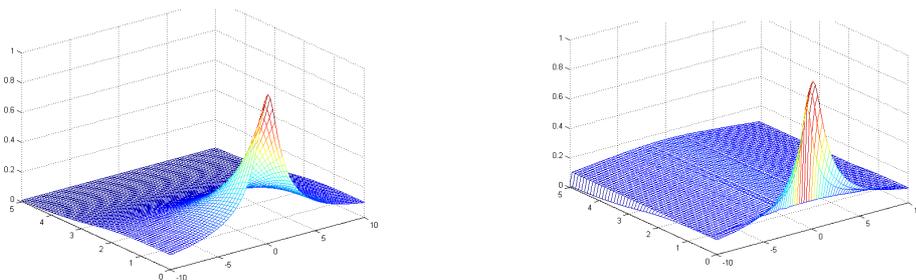


Fig.1. The effective magnetic flux penetration of the at  $n=0$  and  $q=1$  for  $t=0.5$  and  $t=1$ .

Next, we consider the case  $n = 1$  and  $q = 1$  in Fig. 3, on the left the initial condition, on the right the solution for  $t = 0.5$ . Schematically, the evolution of the process of penetration of the magnetic field in the regime of viscous flow of vortices with a power-law dependence  $b(x,t)$  on the exponent  $n$  is shown in Figure 2.

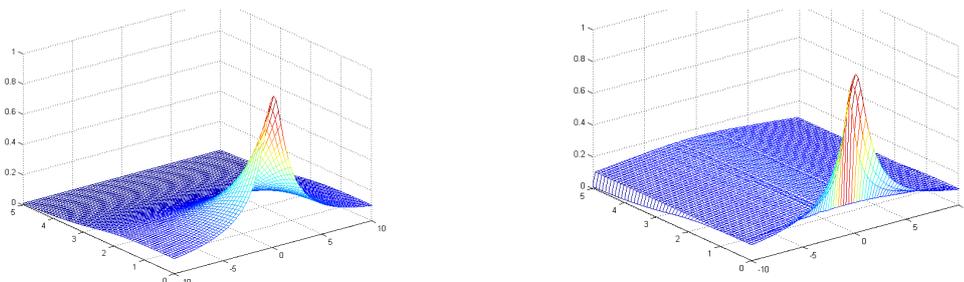


Fig.2. The effective magnetic flux penetration of the at  $n=1$  and  $q=1$  for  $t=1$  and  $t=5$

Next we consider the case  $n = 2$  and  $q = 1$  in Fig. 3, on the left the initial condition, on the right the solution for  $t = 5$ . Schematically, the evolution of the process of penetration of the magnetic field in the regime of viscous flow of vortices with a power-law dependence  $b(x,t)$  on the exponent  $n$  is shown in Figure 3.

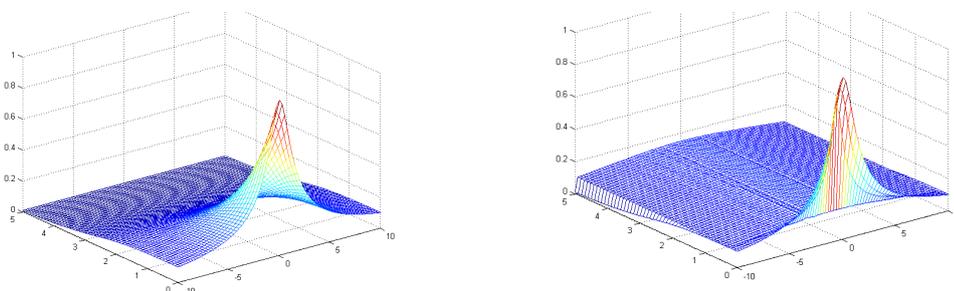


Fig.3. The effective magnetic flux penetration of the at  $n=2$  and  $q=1$  for  $t=5$  and  $t=10$ .

The obtained solution (7) describes the effective penetration of the magnetic flux into the sample, and the magnetic induction is localized in the region between the surface  $x = 0$  and the flux front  $x_p$ . This solution is positive in the plane  $x_p^2 > x^2$  and equals zero outside it. The position of the flow front  $x_p = x(t)$  as a function of time can be described by the relation

$$x_p = z_0 t^{\frac{1}{n+2q}}. \quad (8)$$

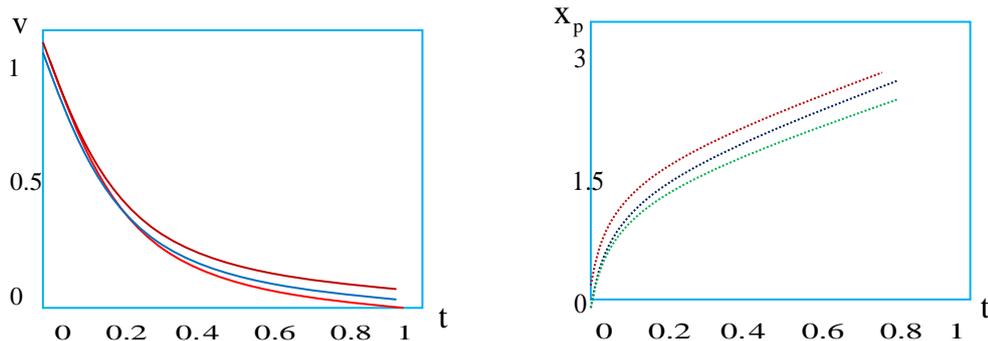


Fig.4. The effective magnetic flux penetration for  $n = 3, 7, 11$ .

The speed of the magnetic flux front decreases rapidly as the magnetic flux propagates (Fig. 4).

$$v_p(t) \sim \frac{dx_p}{dt} \sim t^{\frac{(2q+n-1)}{(n+2q)}}. \quad (9)$$

The spatial and temporal profiles of magnetic flux penetration into the sample depend on a set of three independent parameters,  $n$ ,  $q$ , and  $\alpha$ . It is of interest to consider the non-linear diffusion equation for magnetic induction at different values of the exponents  $n$ ,  $q$  and  $\alpha$ . For a given set of parameters  $n$ ,  $q$  and  $\alpha$  the form of the scaling function  $f(z)$  can be obtained by solving the nonlinear diffusion equation (13) analytically by the self-similar method. Thus, in order to obtain expressions for the spatiotemporal evolution of magnetic induction for various values of the exponents  $n$ ,  $q$ , we will investigate the solution of the diffusion equation. In addition, we analyze the influence of various values of the indicators on the shape of the magnetic flux front in the sample. By varying the parameters of the equation, we can observe different forms of the magnetic flux front in the sample. Note that a similar approach was used in [7] in the framework of nonlinear flow diffusion for the transverse geometry of the sample. As can be shown, different values of the exponents  $n$  and  $q$  generate different spatiotemporal magnetic flux fronts in a superconductor [11].

## CONCLUSION

In summary, we have considered problem of nonlinear diffusion of the magnetic flux injected in an infinite thin type-II superconductor. We have solved it numerically in the most interesting case of flux flow resistivity proportional to the power-law dependence of local magnetic induction  $B$ . The



obtained flux space-time distributions are of the self-similar form with rather striking scaling functions.

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### РОЛЬ ФИЗИКИ В ПРЕПОДАВАНИИ МАТЕРИАЛОВЕДЕНИЯ ТКАНЕЙ

Хаджикаримова Гуласал Таджиалиевна, Усарова Шойрахан

Ферганский государственный университет

**Аннотация:** В данной статье приведены сведения о свойствах и характеристиках тканей, сведения о физико-механических свойствах тканей нового состава. Подчеркнута роль физики в преподавании науки о тканевых материалах.

**Ключевые слова:** модель, анализ, креп, сатин, сочетание, материал, прочность, фурнитура, физические свойства, прочность.

Отделка ткани — это сочетание физико-химических и механических процессов превращения сырой ткани в готовую ткань. Целью отделки тканей является улучшение их внешнего вида и качества. Отделка учитывает химический состав волокон, входящих в состав ткани.

Климатические условия нашего региона показывают, что существует высокая потребность в теплой одежде. Люди используют разную одежду, чтобы защитить себя от естественного дискомфорта. К наиболее распространенной утепляющей одежде относятся пальто, куртки и пальто. Среди них пальто выделяются тем, что их можно носить в разном возрасте и в разных условиях. Основные детали пальто выполнены из тканей разных свойств. Для зимней верхней одежды важны свойства сохранения тепла, водонепроницаемости, воздухопроницаемости и пылепоглощения.